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Secondary Queuing Models for Railway Reservation System with Truncation of the Range of Probability of Surrendering Reserved Tickets.

S. P. Kane* & S. V. Shinkar**

** Vice-Chancellor, Rashtrasant Tukadoji Maharahj Nagpur University, Nagpur (India)*
*** Junior Research Fellow, Department of Statistics, Rashtrasant Tukadoji Maharaj Nagpur University, Nagpur (India)*

ABSTRACT:

The existing queuing theory provides various models which are inadequate to handle "Secondary queues" generated out of the situation of the availability of the 'Service Surrender' facility. The present paper deals with such secondary queues as applied particularly in Indian Railways ticket reservation system.

Keywords: Secondary queues, Service Surrender Facility.

1. INTRODUCTION:

Indian Railways (IR) is the principal mode of transport in the country. It is one of the world's largest rail networks under a single management. The route length is around 63,332 km with more than 8000 stations. As it is the backbone of nation's transport system, IR owns more than 25,000 wagons, 45,000 different types of coaches and 8000 locomotives. The system carries about 5,000 million passengers generating a traffic output of 340 billion passenger kms.

a)Role of Queuing Theory in Indian Railway Reservation System: -

The IR reservation tickets can be obtained by the passengers by approaching to the service counter forming a queue. There are two different modes available for the passengers for getting reservation tickets,

1) By directly approaching to the service counters located at different railway stations.
2) By utilizing internet facility for obtaining online reservation tickets.

In any of the above cases the passengers have to join only one single queue indirectly. There are three special features which can be easily noticed. These are;

1) Once the passenger (customer) receives the service (i.e. confirmed railway reservation tickets) it is possible for the customer to surrender the service (i.e. tickets). Hence the service type is such that it can be surrendered any time.
2) Secondly we noticed that, the queuing system for a particular train starts only after a specific point of time that is say 120 days prior to the date of journey.
3) The date of journey is a time point say 'T' when the entire system collapses. The system starts functioning a units of time briar to this point T. (i.e. we can say that A = 120 for any train and correspond T.)

When the customer approach the service counter and come to know that the confirmed tickets are all sold out, he/she has to join by choice the 'waiting list' queue. As such the formation of queue of waiting list customers starts as and when the earlier customer who got the confirmed reservation ticket surrendered it, the 'waiting list' queue customer get the surrendered service by first come first served way of queue discipline.
This creates uncertainty in the minds of the customer in 'waiting list' about getting the confirmed ticket.
And as such the customers on 'waiting list' now called here onwards as customers in the 'Secondary queue' are a part of such a queue which has the customers without any guarantee of getting the service.
A study of such a queue becomes an interesting problem.
In the present project work we developed some queueing models for 'Secondary queues'. These mathematical models can be used to study some characteristics of the secondary queues.
Hence queueing theory plays an important role in understanding and analyzing the Indian Railway Reservation System.

b) Some Definitions:

Primary queue:
It is the waiting line of the units in front of the service counter before they are registered on the 'waiting list'.

Secondary queue:
It is the waiting line of the units which are registered on the 'waiting list'.
Here we note that a customer can join the secondary queue only if he comes through the primary queue. The customers have an option whether to join the secondary queue or not after having come through the primary queue.
Since we consider only that queue which is formed by the customers who are on the waiting list, we try to find out the waiting time distribution of such customers.
Further, those customers which are registered on waiting list may have already waited for some time in the queue to reach the service counter just only to find that the service is not available and such on an average they have waited for $\frac{p}{\mu\,(1-p)}$ units of time (from equation 4 and assumption 8 of elementary model).

Now since they are registered on the waiting list, they may get the service as and when the previously served customers surrender their services and this second phase of waiting may be for unpredictable time span. Now we attempt to find out this time span. For this, first we define the following:

Service Holding Time (SHT) of the Queuing System:

Service holding time of a queuing system is the average duration of time for which customer holds the service, before it is surrendered.

We can find out the SHT distribution as a probability distribution. We note that SHT is a continuous random variable.

For the sake of analytical convenience we convert SHT in to a discrete random variable as follows.

Let p be the probability that a customer enjoys the service for a unit time. The probability that he surrenders the service some time during a unit time is (1-p).

Therefore probability that a customer completes 't' units of time before he surrenders the service i.e. P(T=t) is as follows:

$$P(T = t) = p^{t-1}(1-p) \qquad t = 1,2,3,\ldots\ldots$$

But this is the probability mass function of a Geometric Distribution. Hence T is distributed as a Geometric variable.

Now since 'p' differs from person to person randomly, taking any value in the range [0,1], it is appropriate to consider it as a random variable. The appropriate distribution of p is generally considered as Beta Distribution with parameters 'a' and 'b' which is given by

$$f(p) = \frac{1}{B(a,b)} p^{a-1}(1-p)^{b-1} a,b > 0;\ 0 < p < 1$$

Where B(a, b) is Beta function with parameters a and b given by

$$B(a\ ,b) = \int_0^1 p^{a-1}\,(1-p)^{b-1}\,dp$$

$$= \frac{\Gamma a \Gamma b}{\Gamma(a+b)}$$

$$= \frac{(a-1)!\,(b-1)!}{(a+b-1)!}$$

Here T is distributed as Geometric and p is distributed as Beta.

Thus according to D. J. Bartholomew(1963), SHT(T) follows a Compound Beta distribution as follows.

$$P[T = t] = \int_0^1 f(p)p^{t-1}(1-p)dp$$

$$= \frac{1}{B(a,b)}\int_0^1 p^{a-1}(1-p)^{b-1}p^{t-1}(1-p)dp$$

$$= \frac{1}{B(a,b)}\int_0^1 p^{a+t-2}(1-p)^{b}dp$$

$$= \frac{1}{B(a,b)} B(a+t-1, b+1)$$

$$= \frac{B(a+t-1, b+1)}{B(a,b)} \qquad a,b > 0; t = 1,2,\ldots,\infty$$

2. (i) Analysis of Model with infinite range of SHT and$\propto 1 \le p \le \propto 2$:

We know that

p = probability that customer do not surrender the service

&

(1-p) = probability the customer surrenders the service

In railway reservation case, generally the customers purchase the tickets with the intensions of not surrendering it except under special circumstances because there is a penalty of surrendering the service. Although the value of p can never be 1, t is reasonably high. Hence p assumes value in the higher range say from $\alpha 1$ to $\alpha 2$ where $\alpha 1$ & $\alpha 2$ are such that $\alpha 1 < \alpha 2$ & $\alpha 2 < 1$. e.g. $\alpha 1 = 0.7$ & $\alpha 2 = 0.90$

Analysis of model with infinite range of SHT and $\alpha 1 \le p \le \alpha 2$: -

Considering a queuing system having SHT ranging from 1 to ∞ and the range of 'p' truncated at both the ends.

Now since

$$f(p) = \frac{1}{B(a,b)} p^{a-1}(1 = p)^{b-1}$$

The distribution of p truncated at both the ends may be given by

$$g(p) = \frac{f(p)}{\int_{\alpha 1}^{\alpha 2} f(p)dp}$$

$$= \frac{\frac{1}{B(a,b)} p^{a-1}(1 = p)^{b-1}}{\int_{\alpha 1}^{\alpha 2} \frac{1}{B(a,b)} p^{a-1}(1 = p)^{b-1}}$$

$$= \frac{p^{a-1}(1 = p)^{b-1}}{\int_{\alpha 1}^{\alpha 2} p^{a-1}(1 = p)^{b-1}}$$

$$= \frac{p^{a-1}(1 = p)^{b-1}}{\left[\int_{0}^{\alpha 2} p^{a-1}(1 = p)^{b-1}\,dp - \int_{0}^{\alpha 1} p^{a-1}(1 = p)^{b-1}dp\right]}$$

$$= \frac{p^{a-1}(1 = p)^{b-1}}{[B_{\alpha 2}(a,b) - B_{\alpha 1}(a,b)]}$$

Probability distribution of SHT: -

Compound geometric doubly truncated Beta distribution has the following probability function

$$P(T = t) = f_t = \int_{\alpha 1}^{\alpha 2} g(p)p^{t-1}(1-p)dp$$

$$= \int_{\alpha 1}^{\alpha 2} \frac{p^{a-1}(1 = p)^{b-1}}{[B_{\alpha 2}(a,b) - B_{\alpha 1}(a,b)]} p^{t-1}(1-p)dp$$

$$= \frac{1}{[B_{\alpha 2}(a,b) - B_{\alpha 1}(a,b)]} \int_{\alpha 1}^{\alpha 2} p^{a+t-2}(1-p)^b dp$$

$$= \frac{1}{[B_{\alpha 2}(a,b) - B_{\alpha 1}(a,b)]} \left[\int_{0}^{\alpha 2} p^{a+t-2}(1-p)^b dp - \int_{0}^{\alpha 1} p^{a+t-2}(1-p)^b dp\right]$$

$$\therefore P(T = t) = f_t = \frac{[B_{\alpha 2}(a+t-1, b+1) - B_{\alpha 1}(a+t-1, b+1)]}{[B_{\alpha 2}(a,b) - B_{\alpha 1}(a,b)]}$$

(ii) Average waiting time of a customer in the secondary queue:

The average duration of utilizing the service by a customer is

$$E(t) = \sum_{t=1}^{\infty} t.f_t$$

Using f_t equation, we get

$$E(t) = \sum_{t=1}^{\infty} t.\frac{[B_{\propto 2}(a+t-1,b+1) - B_{\propto 1}(a+t-1,b+1)]}{[B_{\propto 2}(a,b) - B_{\propto 1}(a,b)]}$$

(iii) Other Characteristics:

The distribution function of T: -

The distribution function of T is given by

$$F_T = \sum_{t=1}^{T} f_t$$

Using f_t equation, we get

$$F_T = \sum_{t=1}^{T} \frac{[B_{\propto 2}(a+t-1,b+1) - B_{\propto 1}(a+t-1,b+1)]}{[B_{\propto 2}(a,b) - B_{\propto 1}(a,b)]}$$

Survival function of the customer: -

The survival function G_T is given by

$G_T = 1 - F_{(1-T)}$ if t is discrete

$$= 1 - \sum_{t=1}^{T-1} \frac{[B_{\propto 2}(a+t-1,b+1) - B_{\propto 1}(a+t-1,b+1)]}{[B_{\propto 2}(a,b) - B_{\propto 1}(a,b)]}$$

Service wastage of the model: -

The service wastage of the model S_T is given by

$$S_T = \frac{f_T}{G_T}$$

$$= \frac{\dfrac{[B_{\propto 2}(a+t-1,b+1) - B_{\propto 1}(a+t-1,b+1)]}{[B_{\propto 2}(a,b) - B_{\propto 1}(a,b)]}}{1 - \sum_{t=1}^{T-1} \dfrac{[B_{\propto 2}(a+t-1,b+1) - B_{\propto 1}(a+t-1,b+1)]}{[B_{\propto 2}(a,b) - B_{\propto 1}(a,b)]}}$$

3. (i) Analysis of Model with finite range of SHT and$\propto 1 \le p \le \propto 2$: -

Let us consider a queuing system that works for some finite time and after that the entire system vanishes. So here SHT ranges from zero to some finite number say ‘A’. In other words we truncate T to the right at A and the range of ‘p’ is truncated at both the ends.

Probability distribution of SHT truncated to the right at T = A

$$P(T=t) = f_t = \begin{cases} \dfrac{K.[B_{\propto 2}(a+t-1,b+1) - B_{\propto 1}(a+t-1,b+1)]}{[B_{\propto 2}(a,b) - B_{\propto 1}(a,b)]} & 0 < t \le A \\ 0 & \text{other wise} \end{cases}$$

$$\because \sum_{t=1}^{\infty} f_t = 1$$

$$\Rightarrow \sum_{t=1}^{A} f_t + \sum_{t=A+1}^{\infty} f_t = 1$$

$$\sum_{t=1}^{\infty} f_t = \frac{\sum_{t=1}^{A} K.\,[B_{\alpha 2}(a+t-1, b+1) - B_{\alpha 1}(a+t-1, b+1)]}{[B_{\alpha 2}(a, b) - B_{\alpha 1}(a, b)]}$$

$$K = \sum_{t=1}^{A} \frac{[B_{\alpha 2}(a, b) - B_{\alpha 1}(a, b)]}{[B_{\alpha 2}(a+t-1, b+1) - B_{\alpha 1}(a+t-1, b+1)]}$$

$$f_t = \sum_{t=1}^{A} \frac{[B_{\alpha 2}(a,b) - B_{\alpha 1}(a,b)]}{[B_{\alpha 2}(a+t-1, b+1) - B_{\alpha 1}(a+t-1, b+1)]} \cdot \left[\frac{[B_{\alpha 2}(a+t-1, b+1) - B_{\alpha 1}(a+t-1, b+1)]}{[B_{\alpha 2}(a,b) - B_{\alpha 1}(a,b)]}\right]$$

(ii) Average waiting time of a customer in the secondary queue:

The average duration of utilizing the service by a customer is

$$E(t) = \sum_{t=1}^{A} t.\,f_t$$

Using f_t equation, we get

$$= \sum_{t=1}^{A} t.\sum_{t=1}^{A} \frac{[B_{\alpha 2}(a,b) - B_{\alpha 1}(a,b)]}{[B_{\alpha 2}(a+t-1, b+1) - B_{\alpha 1}(a+t-1, b+1)]} \cdot \left[\frac{[B_{\alpha 2}(a+t-1, b+1) - B_{\alpha 1}(a+t-1, b+1)]}{[B_{\alpha 2}(a,b) - B_{\alpha 1}(a,b)]}\right]$$

(iii) Other Characteristics:

The distribution function of T: -

The distribution function of T is given by

$$F_T = \sum_{t=1}^{T} f_t$$

Using f_t equation, we get

$$= \sum_{t=1}^{T} \sum_{t=1}^{A} \frac{[B_{\alpha 2}(a,b) - B_{\alpha 1}(a,b)]}{[B_{\alpha 2}(a+t-1, b+1) - B_{\alpha 1}(a+t-1, b+1)]} \cdot \left[\frac{[B_{\alpha 2}(a+t-1, b+1) - B_{\alpha 1}(a+t-1, b+1)]}{[B_{\alpha 2}(a,b) - B_{\alpha 1}(a,b)]}\right]$$

Survival function of the customer: -

The survival function of the customer is given by

$G_T = 1 - F_{(T-1)}$

$$= 1 - \sum_{t=1}^{T-1} \sum_{t=1}^{A} \frac{[B_{\alpha 2}(a,b) - B_{\alpha 1}(a,b)]}{[B_{\alpha 2}(a+t-1, b+1) - B_{\alpha 1}(a+t-1, b+1)]} \cdot \left[\frac{[B_{\alpha 2}(a+t-1, b+1) - B_{\alpha 1}(a+t-1, b+1)]}{[B_{\alpha 2}(a,b) - B_{\alpha 1}(a,b)]}\right]$$

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Service wastage of the model: -

The service wastage of the model S_T is given by

$$S_T = \frac{f_T}{G_T}$$

$$= \frac{\sum_{t=1}^{T}\sum_{t=1}^{A}\frac{[B_{\propto 2}(a,b) - B_{\propto 1}(a,b)]}{[B_{\propto 2}(a+t-1,b+1) - B_{\propto 1}(a+t-1,b+1)]}\cdot\left[\frac{[B_{\propto 2}(a+t-1,b+1) - B_{\propto 1}(a+t-1,b+1)]}{[B_{\propto 2}(a,b) - B_{\propto 1}(a,b)]}\right]}{1-\sum_{t=1}^{T-1}\sum_{t=1}^{A}\frac{[B_{\propto 2}(a,b) - B_{\propto 1}(a,b)]}{[B_{\propto 2}(a+t-1,b+1) - B_{\propto 1}(a+t-1,b+1)]}\cdot\left[\frac{[B_{\propto 2}(a+t-1,b+1) - B_{\propto 1}(a+t-1,b+1)]}{[B_{\propto 2}(a,b) - B_{\propto 1}(a,b)]}\right]}$$

4. PARTIAL SERVICE FACILITY

It is to be noted that in general when the service is of the type that it can be surrendered, the server offers the same surrendered service to the customers of secondary queue. It is of interest to note that the server of Indian Railway reservation system can offer partial services to a few customers of the secondary queue. For example, let the services available with the server to N customers, the server can provide the service to the customer of secondary queue only after earlier customer surrenders the service. In railway reservation system the server now provides partial service by way of allotting RAC tickets to two customers out of one complete surrendered service. Mathematically it can be explained as follows, available services with the server equal to N. before the procedure of surrendering starts and before utilizing all services by the server.

Now one service is surrendered to the server. Instead of giving that surrendered service to the first customer of secondary queue the server gives the same service to two customers partially. As such if M services are surrendered back then from the secondary queue M + R number of customers will get partial service. This concept can be exploited for developing queuing models in this discipline. Many variations can be thought of for developing the corresponding queuing models such as

1) Every surrendered service is broken into pieces for customers of secondary queue.
2) A small proportion of surrendered services are only broken into pieces and so on.
3) We leave this topic of developing the models for such situations as open problem for further research in this area.

5. TABLES:

t	ft				
	b=10		A=50		
	a=5	a=10	a=15	a=20	a=25
2	0.208333	0.238095	0.23077	0.215057	0.198422
4	0.028595	0.062112	0.083028	0.094087	0.09907
6	0.005418	0.018841	0.032639	0.043644	0.051567
8	0.00129	0.006441	0.013819	0.021298	0.02785
10	0.000365	0.002427	0.006232	0.010864	0.015544

t	ft				
	a=10		A=50		
	b=5	b=10	b=15	b=20	b=25

2	0.208388	0.238095	0.230769	0.215054	0.1984127
4	0.089893	0.062112	0.040293	0.026882	0.01862765
6	0.043054	0.018841	0.008429	0.004111	0.002173226
8	0.022366	0.006441	0.002039	0.000741	0.000302889
10	0.012398	0.002427	0.000556	0.000153	0.000049

E (t)										
B \ A	5	10	15	20	25	30	35	40	45	50
5	37.84701	127.3023	266.7709	456.4774	696.3124	986.2144	1326.153	1716.112	2156.085	2646.066
10	65.6190	179.0735	341.6402	554.9439	818.8241	1133.076	1497.565	1912.206	2376.948	2891.757
15	100.4792	238.5530	423.3822	659.1702	946.0564	1283.787	1672.123	2110.887	2599.955	3139.241
20	142.6841	306.4734	513.3200	770.8844	1079.914	1440.264	1851.663	2313.862	2826.665	3389.927
25	192.3154	383.0908	612.0357	891.0179	1221.599	1603.878	2037.629	2522.584	3058.498	3645.172
30	249.4065	468.5066	719.7932	1020.060	1371.832	1775.535	2231.065	2738.179	3296.617	3906.146
35	313.9732	562.7657	836.7178	1158.274	1531.035	1955.820	2432.692	2961.476	3541.927	4173.800
40	386.0241	665.8895	962.8717	1305.801	1699.462	2145.108	2643.004	3193.073	3795.111	4448.883
45	465.5643	777.8890	1098.286	1462.721	1877.265	2343.640	2862.337	3433.397	4056.678	4731.972
50	552.5967	898.7702	1242.979	1629.078	2064.536	2551.572	3090.920	3682.752	4327.004	5023.509
55	647.1234	1028.536	1396.959	1804.897	2261.332	2769.006	3328.907	3941.354	4606.367	5323.830
60	749.1459	1167.189	1560.230	1990.195	2467.688	2996.009	3576.409	4209.357	4894.970	5633.190

T	S_t			
	b=4		A=40	
	a=3	a=13	a=23	a=33

5	0.364105	0.193022	0.134133	0.105044
10	0.252109	0.159334	0.119816	0.098382
15	0.196305	0.139436	0.111729	0.09559
20	0.166774	0.129584	0.109618	0.097292
25	0.155075	0.129877	0.115263	0.105778
30	0.163606	0.146308	0.135593	0.128324
35	0.221749	0.210286	0.20276	0.197444
40	1.000000	1.000000	1.000000	1.000000

T	S_t			
	a=8			A=40
	b=4	b=14	b=24	b=34
5	0.251373	0.5384616	0.6666667	0.7391304
10	0.1942421	0.4516158	0.5853659	0.6666667
15	0.1620491	0.388929	0.5217395	0.6071429
20	0.1449253	0.3418158	0.470599	0.5573776
25	0.1405622	0.3066206	0.4287728	0.5151746
30	0.1538104	0.2864425	0.3961372	0.4795622
35	0.2153554	0.3099358	0.393109	0.4626763
40	1.000000	1.000000	1.000000	1.000000

T	S_t			
	b=3 a=20			
	A=10	A=20	A=30	A=40
5	0.222424	0.129706	0.110766	0.103458
10	1.000000	0.144874	0.107253	0.095282
15		0.208499	0.112366	0.091872
20		1.000000	0.132646	0.093438
25			0.200128	0.102059
30			1.000000	0.12491
35				0.194543
40				1.000000

t	Model III Ft				
	α1=0.1		b=5	α2=0.9	
	a=5	a=10	a=15	a=20	a=25
2	0.227273	0.208333	0.178571	0.153846	0.134409
4	0.061189	0.089869	0.095991	0.094017	0.089351
6	0.020979	0.043043	0.054715	0.059652	0.060969
8	0.008484	0.02236	0.032735	0.039087	0.042568
10	0.00387	0.012395	0.020399	0.026336	0.030332

t	ft				
	α1=0.1 a=5 α2=0.9				
	b=5	b=10	b=15	b=20	b=25

2	0.227273	0.208333	0.178571	0.153846	0.134409
4	0.061189	0.028595	0.014822	0.008547	0.005346
6	0.020979	0.005418	0.001779	0.000707	0.000323
8	0.008484	0.00129	0.000279	0.000078	0.000027
10	0.00387	0.000365	0.000054	0.000011	0.000003

E(t)					
1=0.1			2=0.9		
	a=5	a=10	a=15	a=20	a=25
b=5	0.984614	1.33723	1.535266	1.617728	1.6359
b=10	0.577524	0.930138	1.128142	1.210604	1.228776
b=15	0.429876	0.782492	0.980528	1.06299	1.081162
b=20	0.346856	0.699472	0.897508	0.97997	0.998142
b=25	0.292386	0.645002	0.843038	0.9255	0.940672

t	St				
	T=10 b=5 α1=0.1 α2=0.9				
	a=5	a=10	a=15	a=20	a=25
1	0.429548	0.165682	0.087867	0.054007	0.036279
2	0.088749	0.064719	0.044830	0.031957	0.023595
3	0.022187	0.027097	0.023712	0.019332	0.015576
4	0.006433	0.012043	0.012954	0.011935	0.010427
5	0.002101	0.005638	0.007287	0.007507	0.007072
6	0.000756	0.002763	0.004209	0.004804	0.004855
7	0.000295	0.001409	0.002490	0.003125	0.003371
8	0.000124	0.000746	0.001507	0.002063	0.002367
9	0.000055	0.000407	0.000930	0.001381	0.001678
10	0.000026	0.000229	0.000585	0.000937	0.001202

t	St				
	T=10 a=5 α1=0.1 α2=0.9				
	b=5	b=10	b=15	b=20	b=25
1	0.429548	0.991414	1.538629	2.072319	2.597064
2	0.088749	0.096818	0.087224	0.076639	0.067562
3	0.022187	0.012060	0.006488	0.003785	0.002375
4	0.006433	0.001824	0.000601	0.000237	0.000107
5	0.002101	0.000323	0.000067	0.000018	0.000006
6	0.000756	0.000065	0.000009	0.000002	0.000000
7	0.000295	0.000015	0.000001	0.000000	0.000000
8	0.000124	0.000004	0.000000	0.000000	0.000000
9	0.000055	0.000001	0.000000	0.000000	0.000000
10	0.000026	0.000000	0.000000	0.000000	0.000000

6. CONCLUSIONS:-

Analysis of service surrender queues is dominated mainly by a secondary queue. The main focus is on finding the expected waiting time of a customer in the entire system. The average time taken by a customer to reach the server is E and the average waiting time in the secondary queue is $E(t) = \sum_{t=1}^{\infty} t.\frac{[B_{x2}(a+t-1,b+1)-B_{x1}(a+t-1,b+1)]}{[B_{x2}(a,b)-B_{x1}(a,b)]}$. The value of service wastage S_T is a measure of distribution in the secondary queue. More the value if S_T more are the chances of getting the surrendered service to the new customers.

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