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Perception of Mathematics Teachers on the Use of New Proof of Triangle Inequality

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ABSTRACT

This paper describes the emerging perception from an in-depth case study exploring the Perception of Mathematics Teachers on the Use of the New Proof of Triangle Inequality. The research participants of the study were the selected mathematics teachers of one of the private schools in Pagadian City. This study was based on Sharan B. Merriam's Case. Central research problem was the teachers' impressions of the new proof of the triangle inequality and its usefulness. Data was collected in an interview through a series of focus group discussions. The perceptions of the mathematics teachers about the proof were impressions, difficulties, and the usefulness of the proof. It was found that the teachers' impressions about the proof being an innovation of the established proof, the proof having intriguing applications, and the realizations of the teachers after being introduced to the new proof. The difficulties of the mathematics teachers were the tendency for the students to commit mistakes, difficulty in assigning the sides of the triangle, and difficulty in determining the exact values. Also, The usefulness that described the proof was convenient to use, has applications to other branches of mathematics, and is beneficial to the students. The researcher formulated recommendations based on the mathematics teachers' perceptions.

KEYWORDS: *Triangle Inequality, Mathematics Teachers' Perceptions, Proof*

INTRODUCTION

The basic concepts of Mathematics are discovered through curious minds. It is commonly said in mathematics education that mathematical assertions are either true or untrue. It is also widely known that many students find this concept difficult to grasp. The distinction between common sense and mathematical logic is stressed by many authors and researchers in psychology and mathematics education (Durand-Guerrier, 2008). One of the most fundamental structures for comprehending mathematical truth is the implication (Rodd, 2000), and according to (Hoyles & Kuchemann, 2002), understanding and generating proofs require an appropriate interpretation of implication.

For a theorem to be applicable, it needs to be shown correctly using a proof. Proving is integral to professional mathematical practice (Rav, 1999; Luzano & Ubalde, 2023), and mathematicians use proofs to communicate with one another and with students (Lew et al., 2016). The most significant use of a proof is to establish facts or truth about a statement. Various studies have suggested that proof plays a variety of roles in mathematics. (De Villiers, 1990; Knuth, 2002a): a) to verify that a statement is true, b) to explain why the statement is true, c) to communicate mathematical knowledge, d) to discover or create new mathematics, e) to systematize statements into an axiomatic system. Proof is one of the most vibrant fields of research in mathematics education at present (Rodd, 1998). Being able to establish a concept is not easy.

Many mathematical theories have been discovered; they are turned into theorems when deduced. Theorems have been an essential guiding tool for Mathematicians to generalize a specific idea throughout the years. Many established theorems, such as Pythagoras' and Binomial Theorem, are famous for their daily use. An example of the Pythagorean Theorem that is useful in our daily lives is that it calculates the shortest way possible in traveling. Also, in Binomial Theorem, it is used in expanding binomials quickly.

Another famous theorem is the Triangle Inequality, which states that the sum of a triangle's two sides is greater than its third side. This is commonly taught by learners who are tackling polygons. There are many existing proofs of this in different fields of Mathematics, such as Complex Analysis, Calculus, and Geometry. As of now, only one proof in Euclidean Geometry is known. Unlike the Binomial Theorem and Pythagorean Theorem, the square of the hypotenuse side of a right-angled triangle is equal to the sum of the squares of the other two sides. (Pythagoras, n.d.) which have different proofs in different approaches in their specific fields. With this, the researcher, a co-author of a paper called the "New proofs of triangle Inequalities," contributed an idea that gives another proof for the triangle inequality. It was made to provide another proof and come up with another way of proving the said theorem using circles and ellipses.

Mathematical proof is significant because it may be used to verify, explain, persuade, generate new information, or even synthesize existing knowledge into axiomatic form (Knapp, 2005). It requires a lot of critical thinking and determination so that an idea may become a good foundation for unraveling the universe's secrets. When a mathematician reads a proof to ensure its accuracy, he or she does something unique, unlike reading a newspaper or a book. According to (Prawitz, 2017) proofs in the sense that we are familiar with them from mathematical practice, are based on conclusions. As a result, it appears reasonable to anticipate that the concept of proof can be stated in certain ways in terms of inference. The inference is a concept that can be applied to various situations. On the other hand, make different product preconceptions about inferences and proofs. The purpose of proofs is to prove the existence of theorems. It is not just that the idea of the proof is factual or veridical in the sense that establishing a causal relationship may necessitate a proof in several steps. According to conversations with a few experienced mathematicians, they also want to understand why a mathematical statement is true and the content and place of the proved statement in a larger framework (Pfeiffer, 2010).

According to (Hanna, 1991), the mathematical community's acceptance of an argument as a mathematical proof is based partly on factors that can be considered social rather than logical. Instead of evaluating an argument against any rigor criterion, it could be evaluated based on its author's reputation or how the description of the theorem to be proved fits into existing mathematical knowledge. According to (Selden & Selden, 2003), validation is a skill. It is an unspoken element of the process to conduct a critical assessment of proofs. It is rarely formally taught in the mathematical curriculum. For undergraduate mathematics majors, proof plays an even bigger role. Proof is a major technique for delivering mathematical information in advanced mathematics courses, and students' ability to generate proof is a primary means of evaluating their performance (Weber, 2010). Several stories of mathematics majors' difficulties have surfaced, with many implying that students' inability to create or comprehend proofs is due to a lack of skills (Zhen et al., 2015; Luzano, 2020). According to (Hanna, 1991), the logical validity, a secondary attribute of an argument, is the structural validity of a new theorem's mathematical reasoning, that is, the actual or potential validity of its form, as opposed to its content, is merely a "hygiene factor," a feature recognized as important but taken for granted. Any convincing proof published in a renowned publication is presumed to be genuine in terms of its form or could be made so without harming its content.

A line is considered legitimate if the mathematical community socially agrees upon the justification for the argument. This line, as well as the entire proof, is deemed invalid if the warrant is erroneous. Support is required if an argument's warrant is reasonable but not generally agreed upon by the mathematical community and the proof is said to have a "gap" in it. Validating a proof is needed so that a theorem can be accepted and used. There must also be vital cause to assume that each statement flows from the preceding statements or from other accepted information, i.e., that there is a valid justification for making that statement in this context (Alcock, L., & Weber, K. 2005). As stated by (Weber, & Mejia-Ramos, 2011), if an argument appears in a reputable source with a reliable review procedure, mathematicians are more inclined to accept it as valid.

Mathematics validates by logical reasoning, whereas science confirms through observation. As a result, proofs are at the heart of mathematics, and the distinction between illustrations, conjectures, and proofs should be highlighted. It is important to note that mathematical results are only legitimate when thoroughly proven (Ross, 1998). The researcher aims to know the perceptions of the use of the new proof of the triangle inequality to the mathematics teachers of one of the private sectarian schools in Pagadian City, particularly in the basic and higher education departments. This study’s intention is also to provide an opportunity to develop proving skills and to enable the teachers to inspire students to pursue a higher degree in Mathematics.

FRAMEWORK

The theories anchored in this study are Jean Piaget’s cognitive learning theory, theory of constructivism, and John H. Flavell’s metacognitive theory.

The cognitive learning theory suggests that the learner actively participates in the process. They come to the table with their skills, knowledge, memories, and relevant information they've learned in the past. When learning something new, individuals process and construct their understanding of a topic based on their past experiences and knowledge. In research on proof perceptions and attitudes towards proof and proving wherein, they found that the types of proof perceptions of the prospective teachers had an effect on the process of proving, and their attitudes towards proof and proving could change concerning their proof perception (Tuba & Ovez, 2012). The metacognition theory includes all the processes involved in regulating our thinking. (Boyle et al., 2015) also believed that reasoning and proving should be main activities throughout the K–12 curricula since it has the potential to develop a deeper understanding of mathematics. However, for this to become a reality, teachers need opportunities to learn reasoning and to prove to support their change away from the authoritative perspective, as it contributes to limiting perceptions and misunderstandings about the nature of proof.

The constructivism approach transforms the learners from a passive recipient of information to an active participant in learning process. Supporting the points of these theories, the findings of (Anapa & Samkar, 2010) that the student's opinion on learning methods of proving are not necessary and professional mathematicians can only make that mathematical proofs. It is possible to change this thought if activities that help students gain the proving abilities and exams that measure their proving abilities are included in the elementary and secondary school education systems. Hence, the researcher initiated the idea that teachers should be the foundation of learning proofs, indicating that the teachers should be exposed and hone the craft of proving.

OBJECTIVES:

This study aims to show an alternative way to prove the Triangle Inequality. It seeks to determine the perceptions of the mathematics teachers on using the new proof of triangle inequality. Specifically, this study should answer the following question/queries:

1. What are the mathematics teachers' impressions on the use of the new proof of triangle inequality?
2. What are the difficulties encountered by the mathematics teachers in the use of the new proof of triangle inequality?
3. How do mathematics teachers describe the usefulness of the new proof of triangle inequality?
4. Based on the findings, what realizations can be made?

METHODS AND PROCEDURES

In this study, the case study design was employed. The researcher made use of qualitative research techniques since the goal was to determine the perceptions of the mathematics teachers to validate the researcher's learning materials, known as New Proofs of Triangle Inequality. Employing the Merriam Case Study Model wherein purposive sampling is used.
This study was conducted in one of the private schools in Pagadian City. It is also one of the sectarian schools wherein Basic Education, and Higher Education are offered. The researcher used purposive sampling. The study participants were the mathematics teachers at the selected private school. The reason why the researcher picked the participants is that the study was mainly about proving which is highly grasped by mathematics teachers.

First, the researcher served as the study's instrument. The interview method was one of the tools employed in this investigation. Before obtaining crucial information, the researcher's interview questions were validated by specialists, including research and school officials. The second instrument being used was the researcher's new proof of triangle inequality. The last instrument was the focus group discussion guide; talks and interviews were used in the study to accomplish its principal goals. The focus group discussion was composed of topics that are essential for gathering the necessary data.

The researcher wrote a formal letter to each of their respective school administrators to conduct the study. The letter contains the purpose and solicits approval for the research instrument's administration from the teachers. The researcher then presented the proof using a PowerPoint presentation to the group of mathematics teachers. To check, the researcher gave an activity to the participants. The researcher utilized the focus group discussion approach in gathering the data. After the presentation, the subject was discussed, and the queries were catered to by the researcher. The researcher proceeded to the participants for the interview sessions. Interviews with study participants took place in their free time. Each question was to be answered orally by the research subjects. Throughout the process, an audio recorder was present. The informed consent that was given before the interview stated that the audio recording of the interview procedure had the participants' approval. Before beginning the interview procedure, the researcher took care to obtain the Inform Consent form to confirm that all participants had read and understood the study's terms and conditions. The responses of the participants were gathered and analyzed.

The gathered data from the interview with the selected participants were analyzed based on a systematic coding (breaking down), following the approach Saldana (2012) suggested. It incorporates breaking down all the data into their most minor parts (the codes) and then restructuring and grouping these codes into units or categories known as themes. When the data was coded, it was easy for the researcher to identify the most common responses to the research questions. In this study, the researcher ensured that the school's ethical protocol conducts the study to acquire, interpret, and distribute findings. The formal interview followed the health rules of the Inter-Agency Task Force (IATF), such as wearing a face mask and face shield.

RESULTS AND DISCUSSION

The analysis induced five (5) themes describing the mathematics teachers' perceptions about the new proof of triangle inequality, namely: (a) Mathematics teachers' impressions on the use of triangle inequality, (b) Difficulties encountered by the mathematics teachers, and (c) Usefulness of the proof.

The research participants are categorized into three institutional cohorts to describe whether the experiences as responses are unified or varied. Each group is given codes such as G0# (Group number) to shorten its description and to maintain the confidentiality of the research participants' profiles.

Mathematics teachers' impressions on the use of triangle inequality

Impression refers to the mathematics teachers' general thoughts on the new proof of triangle inequality. The impressions were about the proof being an innovation of the established proof, the proof having intriguing applications, and the realizations of the teachers after being introduced to the new proof.

Innovation. Refers to the new proof as significantly changed in terms of its convenience and improvement from the established proof.

> "*…amazed kayko sir kay nakakita kag laing pamaagi sa pag prove sa triangle inequality, …, na amaze ko sir sa imong gibuhat kay if ako ang mag thesis kay dili kana akong buhaton kay lisod.*" [… I am amazed because you found another way to prove the triangle inequality. Sir, I amazed by what you did because if I had my thesis, I would not pursue that because of how difficult it is.] – G1

"… ang difference nila kay ang new nag gamit ug distance formula ang old kay wala, …, akong gibuhat sir kay gibutang nako akong self as a student, mas sayun jud sabton ang new proof sir and also mas marelate nimo sya sa new technology sir using coordinate geometry." [… their difference is that the new proof uses distance formula, and the old proof doesn't, …, what I did sir was to put myself as a student, the new proof is easier and also you can relate it to new technology using Coordinate Geometry.] – G1

"…kana diayng mga existing nga theorem diay kay naa pajud diay na sila chance nga mainnovate and ma improve or possibly na mailisan kay naa may bag o nga mas convenient, mas realistic dayun at the same time kanang mas sayun sya iapply based sa imong gi introduce na theorem." [… existing theorems have a chance to be innovated and be improved or possibly replaced because there is a more convenient, more realistic, and at the same time easier to apply theorems based on your introduced theorem.] – G2

"… for me, I would prefer the new proof because you can easily visualize the new proof, and it is easily manipulative when it comes to operations. Easy nako ma come up kung true ba ang statement or not, …, Mas dali gyud sya kay ang old definitional kayo unlike sa new nga basic postulates lang ang gamit and less ra nga definition ang need gamiton." [… for me, I would prefer the new proof because you can visualize the new proof, and it is easily manipulative when it comes to operations. I can easily come up if the statement is true or not, …, the new proof is easier because the old proof is very definitional, unlike the new proof that uses only basic postulates and less definition is needed.] – G3

The impressions of the mathematics teachers were asked regarding the new proof. The teachers were amazed and intrigued at the same time because of the new proof. They have stated that the proof is also logically correct, and the new proof is convenient as well. Throughout the years, many concepts have been improved or improvised by contributors of newer ideas. In the context of mathematics, having to provide another way of proving a concept could be beneficial to the newer generations because it provides convenience and it also invites newer perspectives that are open for discussions to verify if the argument holds. There are some proofs that can offer fresh methods for tackling distinct mathematical issues or provide insight into something tangential to the original setting (Hemmi & Löfwall, 2010).

Intriguing. Refers to the new proof as an intriguing topic that aroused the teacher's curiosity.

"… intriguing ang iyang applications, …, ako jung impression niya kay mas klaro sya compared sa mga katong karaan na theorem, makakita kag triangles and circles di pareha sa uban nga ga focus lang mainly sa triangles." [… the applications are intriguing, …, my impression of the new proof is that it is clearer compared to the old one; you can see triangles and circles, unlike other proofs that mainly focus on triangles.] – G3

The new proof was stated to be intriguing in terms of its applications. The proof offers a wide range of possibilities that could open up new ideas into significant applications. According to the proof comprehension model, a comprehensive understanding of proof entails being able to: provide a summary of the proof that highlights its major objectives; use the proof's methods to establish new theorems in different contexts; deconstruct the proof into its component parts or sub-proofs; and apply the general proof's techniques to a particular example (Weber, 2015).

Realizations. Realizations refer to the thoughts acquired by the mathematics teachers after the presentation of the new proof.

> *"... akoa sir kay yes, naa koy realizations sir sa pag substitute kay with regards naman to sa signs, nakarealize ko nga bahala pag gatudlo nako ug math kay mag struggle japun ko sa pagbutang sa signs samot na if butang nako akong tiil sa sa sapatos sa mga bata narealize nako nga sila pod diay mag lisod sad so narealize nako nga patas on pajud nako akong pasensya nga pasabton nako sila ug unsaon nako pagtudlo sa ilaa ang math."* [… yes sir, I have realizations sir in substituting because it was with regards to the signs, I realized that even if I am already teaching mathematics, I still struggle in putting signs, how much more if I put feet on my students' shoes I have realized that they are also struggling therefore I have realized that I should lengthen my patience in making them understand how I teach in math.] – G1

> *"... it's like finding a needle in a haystack pero it doesn't mean lisod imposible na sya, so that tells us that even though it is a difficult process if you really want to prove something or make a new idea or find a new idea you can realize it."* [… it's like finding a needle in a haystack, but it doesn't mean it is difficult, it is impossible, so that tells us that even though it is a difficult process, if you really want to prove something or make a new idea or find a new idea you can realize it.] – G3

Since the participants' perceptions are being gathered, it is obvious to cite the realizations of the participants in connection with the new proof being introduced. Based on the participants' responses, being more patient in delivering instructions is essential to learning, and there are concepts that can be improved for the betterment of learning (Pang-an, et al, 2022; Casanova, et al. (2023). If different proofs of the same result can frequently be found in mathematics, the variety is significantly larger in the setting of classroom mathematics (Rocha, 2019; Luzano, 2023). Hence, with respect to their multiple intelligences, it is useful for them to be introduced to concepts that are associated with different perspectives that suit their needs.

Difficulties encountered by the mathematics teachers

Refers to the difficulties of the mathematics teachers on the use of the new proof of triangle inequality. The difficulties were the tendency for the students to commit mistakes, difficulty in assigning the sides of the triangle, and difficulty in determining the exact values.

Tendency to commit mistakes. Refers to the possible difficulties that could be experienced by the students in using the new proof of triangle inequality. Participants in the group shared the following:

> *"… I think inig tudlo nako sa mga bata kay ang pag plot sa mga points to make the illustrations, …, there is a possible tendency that the students might not get the correct answer because of minor mistakes in simplifying to get the answer, …, need ug guidance ang mga bata sa pag substitute sa mga values."* [… I think when I'll teach this to the students is the plotting of points to make the illustrations, …, there is a possible tendency that the students might not get the correct answer because of minor mistakes in simplifying to get the answer, …, the students need guidance in the substitution of values.] – G1

Having to teach mathematics makes the teachers know what might be the possible difficulties that the students could face (Aranzo, et al., (2023). They have pointed out that a simple mistake could have a domino effect and ruin the results. In this case, since mathematics teachers are practically problem solvers, addressing the difficulties by relating problems to real situations and by having constant practice in solving might just be the step in dealing with the issue. Mathematical experiences that encourage students to present strong arguments in relevant contexts are what is required for the A-level maths curriculum. What we need to do is introduce these experiences in a way that serves as both a goal in and of itself for the vast majority of students who will go on to study other fields, as well as lays the cognitive groundwork for formal proof for the tiny minority of math specialists who will later draw logical conclusions from precise definitions (Tall, n.d).

Difficulty in assigning the sides. Refers to the difficulty in terms of assigning the sides of the triangle.

> *"… ang pag assign sa sides sa triangle, …, libog ang pag input sa negative ug positive nga signs sa values."* [… the assigning of sides of the triangle, …, confusion in inputting the negative and positive signs of the values.] – G2

The teacher participants stated that the assigning of sides of the triangle. This is a common issue that can be avoided through constant practice and being visually accurate. According to information about some schools, some teachers struggle to create mathematical learning models, such as the triangle topic, which can help students become more mathematically literate. Teachers also struggle to contextualize mathematical concepts in their problem-solving activities (Prabawanto & Mulyana, 2017).

Difficulty in determining the exact values. Refers to the difficulty in calculating the exact values upon using the triangle inequality.

> *"… in terms of accuracy kay kinahanglan nimo nga sakto ang imong values para masulod sya sa range, …, process sa solving, …, need jud ug calculator para dili maglisod."* […in terms of accuracy, you need to be able to get the exact values so that it will fit in the range, …, the process of solving, …, you really need a calculator to lessen the difficulty.] – G3

The teacher participants stated that there is an issue in terms of getting accurate results upon using the new proof. Hence, the use of a calculator is needed to determine the exact values for the desired result. Despite some reservations and opposition at all levels, the use of calculators in education is growing. Many people believe that their use in schools is essential given their increased availability and the likelihood that kids will use them in their daily lives for the rest of their lives (Suydam, 1978).

The usefulness of the proof

Usefulness refers to the relevance of the new proof and implies that the use of the new proof has integrations into other lessons of mathematics. The usefulness that described the proof was convenient to use, has applications to other branches of mathematics and is beneficial to the students. Participants in the group shared the following:

Convenient to use. Refers to the convenience being offered by the use of the new proof of triangle inequality.

> *"… convenient sya compared sya old theorem which makes it easy for the students to understand, …, mas sayun pod sya samot na if butangan ug numerical values, …, understandable sya, …, straight forward ra kaayo ang proof kay simple ra kaayo nga postulates ang gamit and definition ra sa circle ang ginaneed, which makes it easy to understand."* [… it is more convenient compared to the old theorem which makes it easy for the students to understand, …, it is also easier especially if numerical values are assigned, …, it is understandable, …, the proof is straight forward because it uses simple postulates and the definition of circles is the only thing needed, which makes it easy to understand.] – G1

The mathematics stated that the use of the new proof of triangle inequality is more convenient rather than the old proof. But the old proof should not be disregarded as the new proof adds a supplement to the established proof. Extended knowledge of the many purposes of proof conveys information about what proof means in mathematical practice. Therefore, awareness of them should be crucial for how academics perceive the discipline (Hemmi & Löfwall, 2010).

Applicable to other branches of mathematics. Refers to the applications of the new proof of triangle inequality to other branches of mathematics.

> *"… magamit sya sa conics nga topic sa precalculus dayun dali rapod sya ma integrate samot nag naa nay background sa geometry ang mga bata daan, …, magamit sya sa Coordinate Geometry, application sa distance formula ug conics, magamit pod sya sa triangles jud pod nga topic.* [… it can be used in conics, a topic in calculus, and it will be easy to integrate, especially if the students have a background in geometry, ..., it can be used in Coordinate Geometry, application of distance formula and conics, it can also be used in the topic of triangles."] – G2

> *"… naay application sa ang theorem sa duha ka concepts, …, ma apply pod nimo sya sa real-life nga if makakita kag duha ka circles kay pwede diay sya maka form ug triangle."* […the theorem has an application to two concepts, …, you can apply it in real-life if you can see two circles it can form a triangle."] – G2

> *"... magamit sya sa congruence nga topic sa Analytic Geometry, ..., magamit sya sa business subjects inig mag use ug venn diagram, ..., magamit sad sya statistics and probability inig mag describe sa probability measures."* [... it can be used in congruence, a topic in geometry, ..., it can be used in business subjects when using venn diagram, ..., it can also be used in statistics and probability when describing the probability measures.] – G3

The participants stated that the proof has many applications in different fields of mathematics. This indicates the significance of the proof in such a way that it is integrable in many contexts. One of the essential aspects of formal mathematics is mathematical proof. The creation of a series of claims using just definitions and prior results, such as deductions, axioms, or theorems, is how most mathematics textbooks describe the process of mathematical proof. Theoretically, a mathematical proof takes place when the evidence is gathered and later viewed as a method of inferring the statement of the theorem from definitions and the given premises. When a proof can be applied as an established result in subsequent theorems without needing to be broken down into its component parts, it is said to have become a concept (Erh-Tsung Chin, 2003).

Beneficial to the students. Refers to the benefits that can be offered through the use of the new proof of triangle inequality.

> *"... ang bag o, although daghan syag kuti2 at least makita jud nimo ang answer at the end. Kay ang old igo lang mag nimo mamata2 and samot na if imo assignan ug value ang sides daan kabalo naka if maka form syag triangles or not. Ang sa new kay dili nimo makita dayun which is beneficial pod kay you have to solve it, and it will trigger the development of critical thinking skills."* [... the new proof, although it takes longer process, at least you can see the answer at the end. Unlike the old proof wherein you can just visualize whether or not it can form triangles, especially if values are assigned. In the new proof, you cannot see it directly whether it forms a triangle or not, which is beneficial because you have to solve it, and it will trigger the development of critical thinking skills.] – G2

As stated by the participants, the new proof has a benefit in terms of the development of their critical thinking skills. Established concepts and algorithms should serve as the foundation for children's creativity and skill development. Children should most definitely be permitted to research various approaches to an algorithm's objective as part of the natural stimulation of exploration and curiosity. However, such research ought to be seen as inspiring, illuminating, and enhancing conventional methods. Both a solid comprehension of the topics and well-learned methods are essential for success in mathematics (Ross, 1998).

CONCLUSION

Studying the perceptions of mathematics teachers wherein a new concept is introduced to them gives another reason for the importance of proving in the classroom. Proving mathematical proof might be a great challenge to teachers and students, but it gives sense to all things. So, it is worth the time and investment. The answer to the question of existence is through proving. Although there are already made concepts, there are still bridges that link one idea to another. Like civilization, ideas are growing. Teachers learn to teach and become bridges for the students to walk on in reaching their future.

Recommendation

Based on the findings and conclusions of the study, the following are recommended:

- ✓ Mathematics teachers may encourage students to use calculators to ensure the accuracy of the results. This will help the students broaden their knowledge of the functions of the calculators.
- ✓ The teachers may use Geogebra in terms of graphing in applying the new proof of triangle inequality.
- ✓ The mathematics teachers may introduce proving mathematical statements to the students with the use of the new proof of triangle inequality.
- ✓ The mathematics instructional supervisors may conduct workshops introducing the new proof of triangle inequality to the mathematics teachers, the school level, and the division level.
- ✓ Future researchers may find studies that could link two concepts in making an innovative idea.

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