
Image Segmentation Using Morse Operators

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ABSTRACT:

Digital image processing remains a challenging domain of programming. This paper introduces a novel fast algorithm for image segmentation using Morse operators for digital planar surfaces. The proposed algorithm is a region growing technique with added topological control and is extremely useful for applications that need proper object description. Experimental results are presented to demonstrate the effectiveness of the proposed system. Results are stimulating, and show that the segmentation algorithm compares very well with other methods.

Key words: *Digital Planar Surfaces, Morse operators, Neighbourhood, Region based Image segmentation*

1. INRODUCTION:

An image is digitized to convert it to a form which can be stored in a computer's memory or on some form of storage media such as a hard disk or CD-ROM. This digitization procedure can be done by a scanner, or by a video camera connected to a frame grabber board in a computer. Once the image has been digitized, it can be image operated upon by various image processing operations.

Image processing operations can be roughly divided into three major categories, Image compression, Image enhancement and restoration, and Measurement extraction. Image compression is familiar to most people [1, 2]. It involves reducing the amount of memory needed to store a digital image. Image defects which could be caused by the digitization process or by faults in the imaging set-up can be corrected using Image enhancement techniques. Once the image is in good condition, the Measurement extraction operations can be used to obtain useful information from the image [3,4].

In the approach to digital planar surfaces introduced here the concept of pixel is defined in terms of cell as opposed to a point, normally taken in the conventional definition of digital topology[5]. This framework is complete with definitions of path hole, neighbourhood and other necessary concepts. These concepts are presented in the following section.

Additionally, morphological operators are defined as a way to realize property preserving construction of Digital Planar Surfaces (DPS). Morse operators sustain control of the Euler characteristic of such surfaces under cell addition during construction [5]. Morse operators for DPS, as well as their related concepts and properties, are introduced in the following section.

This paper takes a close look at a particularly important application namely, image segmentation. Although the subject of segmentation has been largely discussed in the literature, the use of DPS in this context has shown very interesting advantages. The segmentation technique via Morse operators compares very well against any other methods.

2. SYSTEM ANALYSIS:

A square grid G is a decomposition of \mathbb{R}^2 in two dimension cells V_{ij} (grid cells) with the following properties [5].

- (1) Each point (I,j) , where I and j are integers, is contained in one single cell V_{ij} .
- (2) Given (x,y) in \mathbb{R}^2 , (x,y) is in V_{ij} if and only if $\max(|x-i|,|y-j|) \leq 1/2$.

The vertices of G are the points (a,b) in \mathbb{R}^2 such that $|a-i|=1/2$ and $|b-j|=1/2$, for some integers I and j . The edges of G are the segments whose end points (a,b) and (c,d) are the vertices of G , satisfying $|a-c|+|b-d|=1$ [5]. From the definitions above it can be shown that each cell in G is a square with a side of length one centred in a point (I,j) , where I and j are integers.

An 8- connected neighborhood of a cell V_{ij} is the set of cells V_{kl} where $V_{ij} \cap V_{kl}$ is either a vertex or an edge and a 4- connected neighborhood of V_{ij} is the set of cells V_{kl} where $V_{ij} \cap V_{kl}$ is an edge [5]. V_{ij} and V_{kl} are said to be 8 or 4 connected neighbour if V_{ij} is in the 8- or 4 – connected neighborhood of V_{kl} respectively.

- 1) Let S_1, S_2, \dots, S_k be a set of 8 connected disjoint regions of G and $S = S_1 \cup S_2 \cup \dots \cup S_k$. S is defined as a digital planar surface and S_1, S_2, \dots, S_k are the connected components of S .

A hole in a digital planar surface S is a finite 4- connected region H of G satisfying the following conditions [5].

- 1) If $V_{ij} \in H$ then V_{ij} does not belong to S .
- 2) The 4 – neighbours of a cell in H are in S or in H .

Let e be an edge in a digital planar surface S . If $V_{ij} \in S$ and $V_{kl} \in S$, then e is said to be a internal edge of S . If V_{ij} does not belong to S and V_{kl} does not belong to S , then e is said to be a boundary edge of S .

Let $nv(S)$, $ne(S)$, $nf(S)$ and $nh(S)$ be the number of vertices, edges, faces(cells), connected components, and holes of a digital planar surface $S \subset G$, respectively. The Euler characteristic of S is given by

$$X(S) = nv(S) - ne(S) + nf(S) = nc(S) - nh(S) \quad (1)$$

The Euler characteristic is a topological invariant which plays an important role in topology and it has been used mostly in object classification [5].

2.1 MORSE operators for Digital Planar Surfaces

Let S be a digital planar surface. A cell V_{ij} is called adjacent to S if V_{ij} does not belong to S and V_{ij} is an 8-connected neighbour of any 8 cell of S [5].

Let S be a digital planar Surface and V_{ij} an adjacent cell of S . Then V_{ij} may be defined as a [5]

- 1) (-1) – handle of S if all edges of V_{ij} are in S
- 2) 0-handle of S if (a) one vertex; (b) one edge; and (c) either two or three adjacent edges of V_{ij} are in S .

- 3) 1-handle of S if (a) two vertices; (b) one edge and one vertex; (c) two nonadjacent edges; and (d) two adjacent edges and one vertex of V_{ij} are in S.
- 4) 2-handle of S if (a) three vertices and (b) one edge and two vertices of V_{ij} are in S.
- 5) 3-handle of S if four vertices of V_{ij} are in S.

The described system makes use of twelve operators, which are defined below[5].

- 1) MFC (Make Face and Component): initializes a new connected component as a single cell.
- 2) MF(Make Face): equivalent to adding a 0-handle
- 3) MFH (Make Face and Hole): equivalent to adding a 1-handle
- 4) MFKC (Make Face and Kill component): equivalent to adding a 1-handle
- 5) MF2H(Make Face and Two Holes): equivalent to adding a 2-handle
- 6) MFHKC (Make Face, Make Hole and Kill component): equivalent to adding a 2-handle
- 7) MFK2C(Make Face and Kill Two component): equivalent to adding a 2-handle
- 8) MF3H(Make Face and Three Holes): equivalent to adding a 3-handle
- 9) MF2HKC(Make Face, Make 2 Holes and Kill Component): equivalent to adding a 3-handle
- 10) MFHK2C(Make Face, Make Hole and Kill Two Components): equivalent to adding a 3-handle
- 11) MFK3C(Make Face, and Kill 3 component): equivalent to adding a 3-handle
- 12) MFKH(Make Face, Make Kill Hole): equivalent to adding a (-1)-handle

The operators presented above are called direct MORSE operators and are meant to perform object construction. The inverse MORSE operators can be obtained by replacing the symbol M for K and vice-versa. They are used to “de-construct” a surface.

2.2 Segmentation using MORSE operators

Segmentation algorithm used in this paper is based on region growing, which uses MORSE operators to control the DPS topology. The algorithm expands an initial cell (named seed) according to a similarity function until there is no new cell to be attached.

The segmentation algorithm makes use of a similarity measure defined as a function $\varphi: P(U) \times P(U) \rightarrow [0,1]$, i.e., a function which associates each pair of 8-connected regions in $P(U) \times P(U)$ with a real value between 0 and 1[5]. As expected, this measure indicates how similar (according to some parameters) two regions of U are. The value 1 is considered as absolute similarity and 0 as total dissimilarity. An example of a similarity function is given by[5]

$$\varphi(R_1, R_2) = \frac{1}{\exp \frac{(A(R_1) - A(R_2))^2}{\sigma^2}} \quad (2)$$

Where σ^2 is the variance computed over the neighborhood of the chosen initial cell (or seed) the algorithm begins with an 8-connected region S (DPS with one connected component only) selected either automatically or by an interactive process. $A(R_1)$ and $A(R_2)$ are the mean values of R_{in} and R_{out} respectively. The boundary curve of S is then computed and stored in a dynamic list.

The region growing process is as follows[5]: for each cell V_{ij} such that $V_{ij} \cap S \neq \emptyset$, let $N(V_{ij})$ be the 8-connected neighborhood of V_{ij} , $R_{in} = N(V_{ij}) \cap S$ the cells of $N(V_{ij})$ not in S. If $\varphi(R_{in}, R_{out})$ is greater than a threshold, then the cell V_{ij} is added to S by means of a MORSE

operator. For a new region R_{out} calculated as above, the function value $\varphi(R_{in}, R_{out})$, $I=1,2,...,n$, would give a measure of proximity from R_{out} to each R_i . The new region would then be classified in the same way as the region with the larger similarity value.

In order to add V_{ij} to S, an appropriate MORSE operator is used. The MFH operator is employed over V_{ij} and S so as to create a new hole. The distinction between the internal and external boundary curves lies on the fact that they always have opposite orientations. The topology can be controlled when an appropriate set of MORSE operators is applied[5]. For example, if it is necessary to obtain an object with Euler characteristic equal to 1, all internal curves have to be discarded. To have Euler characteristic equal to 0, only one internal curve is stored. The pseudo code of the segmentation algorithm using MORSE operator is described in algorithm 1[5].

Algorithm 1

1. Input the Image
 - 1.1 Read the pixel value of the image
2. Display the image
3. Select the seed points(a,b)
 - 3.1 Interactively select the seed points row and column vector
4. Initialize the array aa,ab,ac,ad1
5. Find the 8 connectively neighborhood of the seed point (a,b)
6. Update the faces of the DPS
 - 6.1 Make $ab(I,j)=1 \quad \forall i,j \text{ in the DPS}$
7. Update the vertices of the DPS
 - 7.1 $v=v+1$, where v is the number of vertices
8. Update the edges of the DPS
 - 8.1 $ed=ed+1$, where ed is the number of edges
9. Construct the boundary edges of the existing DPS
10. Repeat the following steps until the closed contour is formed
 - 10.1 Select a pixel on the boundary edges of the DPS
 - 10.2 Find the 8 connectivity neighborhood of that pixel
 - 10.3 Identify the common pixels of the 8 connectivity neighborhood of the pixel which is to be tested, with the DPS
 - 10.4 Calculate R_{in} mean
 - 10.5 Calculate R_{out} mean
 - 10.6 Find the value of φ using equation 2
 - 10.7 If $\varphi > threshold$ value, include the pixel under the DPS using MORSE operator.
 - 10.7.1 Update the edges, vertices and edges of the DPS
 - 10.7.2 Construct the boundary edges of the DPS
 - 10.7.3 Go to step 10
 - Else
 - Go to step 10
11. Display the selected pixel position with intensity “0”

3. COMPUTATIONAL EXPERIMENTS:

The segmentation algorithm was tested with the images specified in Fig. 1 and 3. The threshold value is set as 0.3. The seed point coordinates are selected interactively and fed to the program. The segmented part of the input images for the Fig. 1, 3 and 5 is shown in Fig. 2 and 4.

DPS will define border and interior. When segmentation is complete, the topology of the defined objects will have been completely described. In general, there is no geometric description of the objects, a segmentation process is looking for. Topology is therefore the strongest feature that can be explored to correctly detect and describe an object.

Many segmentation procedures resemble surface reconstruction via local methods, with all its usual drawbacks. But the topological approach will form the object in closed regions and orientation is intrinsic of the process. The topological approach is inherently a closed curve segmentation process and defines contour and region of all detected objects in a single pass.



Figure 1. Butterfly image

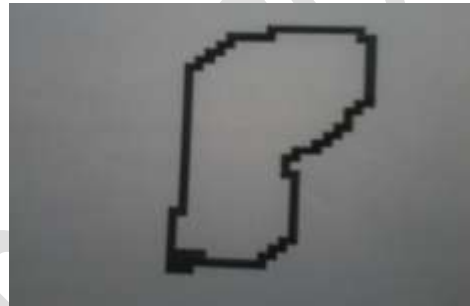


Figure 2. Segmented output of Butterfly image



Figure 3. Leaf image

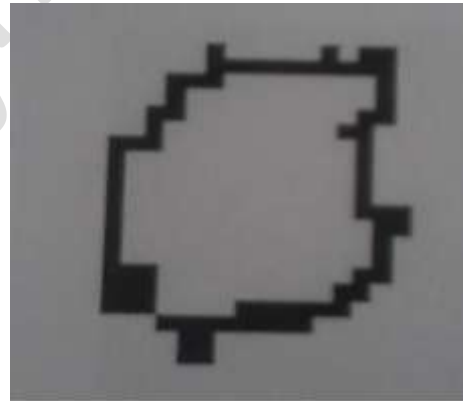


Figure 4. Segmented output of leaf image

4. CONCLUSIONS:

This paper has presented the definition of Digital Planar Surfaces, which differs from the usual concept of digital topology employed in similar classes of application. The main differences lie on the mathematical characterization of pixel, region, hole, boundary, neighbourhood and related entities. The important advantage of the cell approach over traditional ones is that the computation of topological invariants such as Euler characteristic is straightforward. If the pixels were seen as point elements, the calculation of such

invariance would require the construction of polyhedral sets from points, increasing computational costs.

5. REFERENCES :

- i Andrzej J, Alaa M.Hamdy, Segmentation based on homomorphic filtering and improved seeded region growing for mobile robots tracking in image sequences, Source machine graphics & Vision , International Journal archive, 10(4): 2001, 447-466
- ii Barlaud M, Jehan Besson S, Gastaud M, Region based active contours using geometrical and statistical features for Image segmentation, International Conference on Image Processing, 2003, II - 643-6 vol.3.
- iii Jong Bae, Hang Joon, Efficient region based motion segmentation for a video monitoring system, Pattern Recognition Letters 24(1-3), 2003, 113-128
- iv Stephanie Jehan Besson, Michel Barlaud, Video object segmentation using Eulerian region based active contours, International Conference in Computer Vision, Vancouver, Canada, juillet 2001
- v Rosane Minghim, Luis Gustavo, Antonio Castelo, Joao Batista, Morse operators for digital planar surfaces and their application to image segmentation, IEEE Transactions on Image Processing, 13(2),2004