

The Interplay of Conceptual Understanding and Problem-Solving Competence in Mathematics

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ABSTRACT

This study described and evaluated the interplay of students' conceptual understanding and problem-solving competence in mathematics of Grade 11 students in Saint Columban College - Senior High School Department, Pagadian City who were enrolled in General Mathematics.

The study employed a descriptive research design where structured survey questionnaires were used and administered to four (4) strands of Grade 11 namely: ABM, GAS, STEM, and HUMSS.

Findings revealed that the level of students' conceptual understanding and problem-solving competence in mathematics in the four strands in terms of the topics; Function, Rational Function, Inverse Function, and Exponential Function were described as Good and Poor respectively. The test of significant differences in the level of conceptual understanding and problem-solving competence among the four strands yielded an insignificant value of 0.387 and 0.990 respectively, establishing an independent and unique identity level of the four strands in understanding concepts and solving problems in mathematics. In terms of the correlation of the two variables, they yielded a Pearson "r" value of -0.0135 interpreted as "negligible or weakly negative" correlation thus, conceptual understanding and competence in problem-solving may be given equal importance by mathematics teachers as they are expected to interplay one another. The study recommends that in order to attain instructional goals in mathematics, teachers may innovate the packaging of introducing mathematics concepts to students to be effective in solving problems academically and practically.

KEYWORDS: conceptual understanding, problem-solving competence, mathematics

INTRODUCTION

One of the major concerns of educators and parents nowadays is the decline in student achievement. An area of particular concern is science education; where the higher-level thinking skills and problem-solving techniques necessary for success in the technological society need to be developed (Smith & Hausafus, 1998).

Sudaria (2005) described that there was an increasing deterioration of the quality of education in the country, the increasing number of illiterates, most especially in rural areas, and the low performance of students in the fields of language comprehension, mathematics, and science. Thus, it is important that the students would be equipped in understanding the concepts in Mathematics and be aware of their importance to life.



Schoenfeld (1992) has found that in solving mathematical problems, students rely on their knowledge of standard textbook patterns of problem presentation rather than on their knowledge of problem-solving strategies or the intrinsic properties of the problems themselves. Thus, conceptual understanding must serve the purpose as it refers to the ability to comprehend mathematics concepts to perform operations and relate concepts.

In order for students to be successful in mathematics, they must find the meaning of a mathematical problem and look for approaches to a possible solution. Students must analyze and make conjectures about the information guided by the teacher. Hence, conceptual understanding is needed for the students to be mathematically proficient (Bay, 2000).

Polya (2000) explicated that once the problem solver understood the problem, they moved to devise a plan. The problem solver should take all the information in the understanding of the problem combined with personal experiences and devise a plan to efficiently reach the solution.

Rubio (2007) explained that a step-by-step methodical approach provides students with the prerequisite knowledge to learn mathematics efficiently. Moreover, this approach gives students the opportunity to focus on conceptual understanding, computational fluency, and strategic competence. Once mastery of each component skill has been achieved, students are rewarded for their accomplishment and moved ahead to the next skill in sequence. This approach ensures the essential foundation skills such as conceptual understanding and solving problems.

According to De Walle (2007), every teacher of mathematics, whether at the elementary, middle, or high school, has an individual goal to provide students with the knowledge and understanding of the mathematics necessary to function in a world very dependent upon the application of mathematics. Instructionally, this goal translates conceptual understanding that involves the understanding of mathematical ideas and procedures including the knowledge of basic arithmetic facts. Students used conceptual understanding of mathematics when they identify and apply principles, know and apply facts and definitions, and compare and contrast related concepts.

Many teachers used different techniques in teaching mathematics problem solving for the students to understand and would be able to solve. Many students, however, just do the computation on the problem without a sufficient understanding of the concepts (de Walle, 2007). Through this study, the students will be evaluated on their understanding of mathematical concepts and problem-solving competence in mathematics.

Using Piaget's theories, it is the teacher's role to establish a mathematical environment to enable students to construct mathematical knowledge. This environment would provide students with opportunities to hypothesize, test out their thinking, manipulate materials, and communicate their understanding in order to build mathematical knowledge.

With the current trends, problem-solving is central to most current definitions of mathematics literacy. A primary goal of mathematics teaching and learning is to develop the ability to solve a wide variety of complex mathematics problems. These had occupied a central place in the school of mathematics curriculum since antiquity. Only recently that mathematics educators accepted the idea that the development of problem-solving ability deserves special attention. The term problem solving has become the slogan encompassing different views of



what education is, what schooling is, and why we should teach mathematics in general and problem-solving in particular (Schoenfeld, 1992).

The National Council of Teachers of Mathematics (NCTM, 2000) has placed problemsolving first in the list of the process standard. Problem-solving is the key to the development of the individual's critical thinking which is the ultimate goal of mathematics education.

Given the current centrality of problem-solving in mathematics education, much has been written about the importance of students discussing the nature of a problem, the strategies they used to solve the problem, and the reasons why the problem was solved in such ways. In other words, if students know why a certain approach works rather than just what to do, they become better problem solvers.

Supporting theories and practices suggest that when students begin to solve mathematical problems, they should be encouraged to use their own approaches in finding and recording their solutions. Teachers should develop algorithms in a form of instructional materials to enable students to generalize problem-solving steps beyond a single problem and gain confidence in their own power of reasoning as creative processes in order to capture authentic learning in mathematics (Luzano, 2020).

After all the readings and research regarding the correlation between conceptual and problem–solving competence, the researcher found out that a thorough understanding of mathematical concepts influenced the performance of a student in solving mathematical problems, otherwise, it yields poor performance in solving mathematical problems.

Thus, this study would like to describe and evaluate the concrete and non-concrete relationship between conceptual understanding and problem-solving competence in Mathematics among students in Senior High School.

FRAMEWORK

The study is anchored on the notions stipulated in the Framework for Philippine Mathematics Teacher Education that the major goal for students to achieve quality performance in Mathematics is to develop a basic understanding of concepts and procedural skills and the ability to use mathematical reasoning to solve mathematical problems and vice versa since learning occurs whether inductively or deductively.

Illustrated in Figure 1 is the binding relationship between conceptual understanding and problem-solving competence as to their effects on the students of the four strands namely; ABM (Accountancy, Business, and Management), GAS (General Academic Strand), STEM (Science Technology, Engineering, and Mathematics), and (HUMSS) Humanities and Social Sciences.



and Studies



Figure 1. Schematic Diagram of the Study

THE PROBLEM

Statement of the Problem

This study described and evaluated the interplay of students' conceptual understanding and problem-solving competence in mathematics of Grade 11 students in Saint Columban College - Senior High School Department, Pagadian City who were enrolled in General Mathematics.

Specifically, it sought to answer the following queries:

1. What is the level of mathematics conceptual understanding of ABM, GAS, STEM, and HUMSS students on Function, Rational Function, Exponential, and Inverse Function?

2. Is there a significant difference in the level of mathematics conceptual understanding among ABM, GAS, STEM, and HUMSS students?

3. What is the level of mathematics problem–solving competence of ABM, GAS, STEM, and HUMSS students on Function, Rational Function, Exponential, and Inverse Function?

4. Is there a significant difference in the level of mathematics problem–solving competence among ABM, GAS, STEM, and HUMSS students?

5. Is there a significant relationship between conceptual understanding and problem–solving competence in mathematics?

Hypotheses

The following hypotheses shall be tested using the 0.05 level of significance:

1. There is no significant difference in the level of mathematics conceptual understanding among ABM, GAS, STEM, and HUMSS students.



2. There is no significant difference in the level of mathematics problem–solving competence among ABM, GAS, STEM, and HUMSS students.

3. There is no significant relationship between conceptual understanding and problem–solving competence in mathematics.

Significance of the Study

The study may help mathematics teachers improve their competencies in innovating how to facilitate conceptual understanding and problem–solving competence in mathematics.

RESEARCH METHODS AND PROCEDURES

Methods

The research utilized a descriptive method to describe and evaluate the interplay of students' conceptual understanding and problem-solving competence in mathematics. The mode of generating the data for the variables was by soliciting information from a set of participants through testing. Descriptive research is used to describe systematically the facts and characteristics of a given population or area of interest, factually and accurately (Rubio, 2007). Steele and Arth (1998) stated that when the task of the research is to determine the present situation of the variables under investigation, the most appropriate research design is descriptive research.

Locale

This study was conducted at Saint Columban College, Pagadian City Zamboanga del Sur. It is a private educational institution run by the Roman Catholic Diocese of Pagadian. The school is the largest among the Diocesan Schools in Pagadian. The college campus houses the undergraduate programs and the Senior High School.

Participants

The participants of the study are the 105 Grade-11 students of Saint Columban College-Senior High School Department from the strands of ABM, GAS, STEM, and HUMSS. The study used the simple random sampling technique in choosing the participants.

Instrument

The instrument used in gathering data for this study was the test questionnaire. The questionnaire was crafted based on the competencies explicated in the Curriculum Guide of General Mathematics of Senior High School under the K to 12 Curriculum. First Quarter topics were considered, specifically on the Introduction of Function, Rational Function, Exponential Function, and Inverse Function. It was composed of two parts. The first is the 30 – item multiple choice in order to capture and evaluate conceptual understanding in mathematics. The second part is the 10-item problem-solving in order to generate results in measuring students' problem-solving competence in mathematics. The students are given justifiable time in answering the said test.

Data Gathering Procedure

The data gathering procedure of the study involved the following activities which were properly done to generate the necessary data, securing a permit from the principal, then to the adviser of the concerned participants in order to administer the questionnaires. The researcher



also explained the purpose of the study, the manner of answering the test questionnaire, and the retrieval of the accomplished questionnaires.

Statistical Treatment

The data has been treated statistically using the following processes, descriptive statistics like frequency count and mean percentage score (MPS) to measure central tendencies. Analysis of Variance (ANOVA) has been used to measure the significant differences among the four strands concerning the two variables. Pearson "r" was utilized also to test if there is a significant relationship between conceptual understanding and problem–solving competence in mathematics.

RESULTS AND DISCUSSION

Conceptual Understanding in Mathematics

As indicated in Table 1, the overall mean percentage score (MPS) in the four strands was 53.17, interpreted as *Good*. The table also presents the highest MPS of 63.36, interpreted as *Good*, gained from Inverse Function, and the lowest MPS of 44.63, interpreted as *Poor*, gained from Exponential Function. The table shows also that the ABM strand gets the highest MPS of 59.41 (*Good*) and the GAS has the lowest with an MPS of 48.49, interpreted as *Poor*. These findings indicated that the students in the four strands have a good level of conceptual understanding in terms of function, rational, inverse, and exponential functions.

The findings inferred that the students have undergone considerable exposure in several ways to understand mathematical concepts. Steele and Arth (1998) stressed the role of teachers in the teaching of mathematics. Teachers have the responsibility not only to help students to learn the content but also to prepare them for their roles as critical thinkers in today's information age. For students to enhance their life chances, they need first to see some value in mathematics. In addition, they need to become confident in their ability to do the basics and foundations of mathematics. They need "mathematics self-esteem" as reflected in the results where students became not confident in their answers. They need to understand mathematics in a way that gives them the power to make sense of the world around them in order to build up mathematics learning.

Strand/	Function	Rational	Exponential	Inverse	Average	Descriptive
Topics		Function	Function	Function		Equivalent
ABM	54.94	68.69	42.48	71.51	59.41	Good
GAS	42.52	49.97	42.27	59.20	48.49	Poor
STEM	52.43	59.68	42.38	66.77	55.32	Good
HUMSS	42.59	47.90	51.38	55.95	49.46	Poor
MPS	48.12	56.56	44.63	63.36	53.17	Good
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Table 1. Level of Conceptual Understanding in Mathematics of the Four Strands

Hypothetical MPS Range: 76 – 100 – *Very Good*; 51 – 75 – *Good*; 26 – 50 – *Poor*; 0 – 25 – *Very Poor*

As shown in Table 2, the statistical parameters yielded an F-stat value of 1.100 and a probability value of 0.387 which was higher than the 0.05 level of significance, which accepted the hypothesis and established no significant difference in the level of students' conceptual understanding in mathematics. This implies that the four strands would have been more likely same concern towards developing knowledge in mathematics. Thus, an effective



strategy may be explored and later on employed in the classroom to foster improvement in their achievement in mathematics.

Table 2. Test of Significant Difference in the Level of Conceptual Understanding in Mathematics of the Four Strands

Statistical Parameters	Findings
F-Stat	1.100
Probability	0.387
Decision of the Hypothesis	Accepted
Interpretation	No Significant Difference

Problem–Solving Competence in Mathematics

Table 3 showed the data generated for the problem–solving competence of students in the four strands. As indicated in the table, the mean percentage score (MPS) in students' mathematics problem-solving competence is 41.29, interpreted as *Poor*. The table also presented that the highest MPS of 42.45, interpreted as *Poor*, was garnered by grade 11 students in STEM, and the lowest MPS of 40.25, interpreted as *Poor*, was posted for students in GAS.

The findings indicated that the students in the four strands were *Poor* in terms of their mathematics problem-solving competence. The findings infer that there were existing gaps on the side of the teacher and the students. Mathematics teachers may employ different problem–solving strategies in the classroom. It has also been inferred that the students have not undergone considerable exposure to several ways of solving mathematical problems.

De Walle, (2007) argued that the successful accomplishment of activities that comprise good problem-solving requires the use of a variety of strategies or skills. Although there was no list of such skills, it is possible to develop the ability to solve a wide variety of complex mathematics problems.

Strand	Mean	Descriptive Equivalent
ABM	41.41	Poor
GAS	40.25	Poor
STEM	42.45	Poor
HUMSS	41.05	Poor
MPS	41.29	Poor

 Table 3. Level of Problem-Solving Competence in Mathematics of the Four Strands

Hypothetical MPS Range: 76 – 100 – *Very Good*; 51 – 75 – *Good*; 26 – 50 – *Poor*; 0 – 25 – *Very Poor*

As shown in Table 4, the statistical parameters yielded an F-stat value of 0.038 and a probability value of 0.990 which was higher than the 0.05 level of significance, which accepted the hypothesis and established no significant difference in the level of students' problem-solving competence among the four strands. This signifies that the four strands may have the same needs in the aspect of problem-solving. Thus, a call for innovation in the teaching practices of the teacher in facilitating students to be motivated to solve mathematical problems.



Table 4. Test of Significant Difference	in the Level of	Problem–Solving	Competence in
Mathematics of the Four Strands			
Statistical Parameters	Findings		
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Statistical Parameters	Findings
F-Stat	0.038
Probability	0.990
Decision of the Hypothesis	Accepted
Interpretation	No Significant Difference

As shown in Table 5, the testing yielded a Pearson "r" value of -0.0135 and a probability value of 0.865 which is higher than the 0.05 level of significance, which accepted the hypothesis and established no significant relationship between the students' conceptual understanding and problem–solving competence in mathematics.

The findings indicated that the two variables were not significantly correlated. This showed that the conceptual understanding of students bracketed its own dimension in learning and problem-solving competence demarcated its unique form of learning identity. However, these two variables might meet if they are addressed coherently and practically.

Table 5. Test of Significant Relationship between Conceptual Understanding and ProblemSolving Competence in Mathematics

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Statistical Parameters	Findings
Pearson "r"	- 0.0135
Probability Value	0.865
Decision of the Hypothesis	Accepted
Interpretation	No Significant Relationship

Steele and Arth (1998) stressed that when teachers ask students to explain how they think about problems, they are helping them understand that working out mathematics problems is a cycle of thinking, developing, proving, and evaluating a theory or solution to a problem. Students must learn that there are various ways to think about finding solutions. Nonetheless, teachers should provide opportunities for students to think of the number of ways in which mathematical ideas can be appropriately presented. This means problems, concepts, or procedures being learned should have meaning for each student.

Thus, to better endorse 21st Century Learning, students must actively construct their own knowledge with understanding in order to solve academic and practical mathematical problems.

Luzano's Proposition on Mathematical Learning Experience

"Some students may discover concepts to solve problems and some solve problems to discover concepts."

CONCLUSIONS AND RECOMMENDATIONS

The study found a new perspective on conceptual understanding and problem-solving which revealed no significant relationship. The results revealed a mishap in the instructional practices and students' exposure to mathematics due to unsatisfactory MPS. Thus, a theory exists that aside from instructional and student learning factors, there might be a "force for



good" motivation to probably discover concepts to solve problems or solve problems first to appreciate the concepts better.

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