International Journal of Multidisciplinary Approach and Studies
ISSN NO:: 2348 – 537X

Convergence Theorem for Junck-Ishikawa Iteration in Banach Space

Dr Rahul Dwivedi

Sita Ram Samarpan Mahavidyalay, Naraini, Affiliated to Bundelkhand University, Jhansi

ABSTRACT

In this paper, we establish the week convergence the sequence of ishikawa iteration of quasi-nonexpansive maps in the framework of uniformly convex Banach space. The result obtained generalize some well known existing results.

M.S.C. –47H10, 47H09, 47H20, 54E25

Keywords – *Mann iteration, I – quasi- nonexpansive map*

1. INTRODUCTION

Compatible maps and generalization of commuting maps are characterized. In term of coincidence points and common fixed point theorem for commuting maps. G erald Junck introduced this type of iteration in 1988. We use nonexpansive and quasi- nonexpansive mapping in the Junck iteration. We remark that the class of quasi- nonexpansive maps properly includes the class of nonexpansive maps with F(T) $\neq\Phi$ [1] . Gosh and Debnath [2] studied the convergence of iterates of the family of nonexpansive mapping in a uniformly convex Banach space. Rhoades and Temir [3] established the weak convergence of the sequence of the Mann iterates to a common fixed point of T and I by considering the map T to be I- nonexpansive.

Recently kizilton c and Ozdemir established the week convergence of the sequence of modified Ishikawa iterates to a common fixed point of T and I. kuman, Kumethong and Jewwaiworn [4] also established the week convergence for an I- nonexpansive mapping in Banach space. Our aim is to establish the weak convergence of the sequence of Mann iteration to a common fixed point of two maps T and I.

The Mann iteration scheme [5], for n= 0, 1, 2 …………………. and α_2c[01] is defined as

$$X_{n+1} = (1-\alpha_n)\, x_n + \alpha_n T x_n \qquad (1)$$

Further these iterative schemes are developed by taking two mapping S, T : Y $\xrightarrow{x}$

where T(Y) $\subseteq$ S (Y) and $x_0 \in$ Y. Singh et at [6] discuss the following iterative procedure.

$$Sx_{n+1} = f\,(T, x_n)\,,\ n = 0, 1, \ldots \qquad (2)$$

It is called Junek iterative procedures [7]. It f (T, x_n) in [8,9] is replaced byTx_n

$$(1-\alpha_n)Sx_n + \alpha_n Tx_n \qquad (3),$$

It becomes Junck Picard and JunckMann iterationa and ishikawa iteration

$.Sx_{n+1} = (1-\alpha_n)Sx_n + \alpha_n T\gamma_n$ (4)

$S\gamma_{n=}\ (1-\beta_n)Sx_n + \beta_n Tx_n$

2. PRELIMINARIES

Let K be a closed convexbanded subset of a uniformly concave Branch space (X,II.II) and T be a self mapping of X. T is nonexpansive on K if for all x,y ∈K we have.

$IITx\text{-}TyII \leq IIx\text{-}yII$ (5)

A point of f ∈ K is a fixed point of T if Tf =f. We denote the set of the fixed points of T by f (T), where

f (T) ={f ∈ K : T**f** =f}

A map T satisfying

$IITx\text{-}fII \leq IIx\text{-}fII$ (6)

X∈K and f ∈ F (T), is called a quasi- nonexpansive mapping.

Definition 2.1

T is called I- nonexpansive map on K if IITx-TyII ≤ IITx-Ty,for all x,y, ∈K T is called I-quasi nonexpansive map on K if IITx-fII ≤ IITx-fII for all x,y∈K is a common fixed point of I and T if x=Ix==Tx=Sx.

2. MAIN RESULT

Theorem

Let K be a closed, convex and bounded subset of a uniformly convex banach space, and let T and I non self mapping of K with T be an I—quasi-nonexpansive and I a nonexpansive on K. Then x_0∈ K the sequence $\{x_n\}$ of iterates defined by (4) converges weakly to common fixed point of F (T)∩F(S).

Proof

If F(T) ∩F(S) ≠∅ we will assume, and F(T)∩F(S) is not a singleton.

Since, $IISx_{x+1} - f\ II = II\ (I\text{-}\alpha_n)\ (Sx_n + \alpha n\ T\gamma_n) - fII$

$= II\ (I\text{-}\alpha_n)\ (Sx_n\text{-}f) + \alpha_n\ (T\gamma_n\text{-}f)II$

$= II\ (I\text{-}\ \alpha_n)\ (Ix_n\text{-}f) + \alpha_n\ (S\gamma_n\text{-}f)II$

$= II\ (I\text{-}\ \alpha_n)\ (x_n\text{-}f) + \alpha_n\ ((1\text{-}\beta_n)Sx_n + \beta_n\ Tx_n\ \text{-}f)II$

$= (I\text{-}\ \alpha_n)\ IIx_n\text{-}fII + (\alpha_n\ (1\text{-}\beta_n)IISx_n\text{-}fII + \alpha_n\beta_n II\ Tx_n\ \text{-}f\ II$

$= (I\text{-}\ \alpha_n)\ II\ x_n\text{-}f\ II + (\ \alpha_n\ \ (1\text{-}\beta_n)\ IIIx_n\text{-}fII + \alpha_n\beta_n IIIx_n\ \text{-}f\ II)$

$=(I\text{-}\ \alpha_n + \alpha_n\text{-}\alpha_n\beta_n + \ \alpha_n\beta_n)II\ x_n\text{-}f\ II$

$II\ Sn_{n+1} - f\ II = II\ x_n - f\ II$

$\alpha_n \neq o$, {IIxn-fII } is a nonincreasing sequence.

Then

lim II x_n – fII exist

[6-] when Y=X and S = id = I is the identity operator on X.

Example :

To solve cubic equation $x^3+4x^2-5x-10=0$ we rewrite the equation by splititing it into two parts Sx=5x and $Tx=x^3+4x^2-10$.

Following table illustrates the convergence of the iterative scheme.

n	Sx_{n+1}	Tx_n	x_{n+1}
0	-4	-5	-0.8
1	-7.5568	-7.952	-1.51136
2	-4.63956	-4.31543	-0.927913
3	-7.08334	-7.35487	-1.41667
4	-5.04219	-4.81539	-1.00844
-	-	-	-
-	-	-	-
89	-5.98443	-5.98444	-1.19689
90	-5.98443	-5.98443	-1.19689

If α=0.9 then exist the result if α=0.8 then the following table exist also same result

n	Sx_{n+1}	Tx_n	x_{n+1}
0	-3	-5	-0.6
1	-7.6208	-8.776	-1.5241
2	-4.9361	-4.2702	-0.9872
3	-6.639	-7.06482	-1.3278
4	-5.558816	-5.28876	-1.11176
5	-6.256033	-6.4304164	-1.2512066
6	-5.8085998	-5.696798	-1.16172
7	-6.0973008	-6169476	-1.2194602
8	-5.91154557	-5.8651071	-1.1823091
9	-6.0313311	-6.0612776	-1.206266
10	-5.954186	-5.9349	-1.1908372
11	-6.0039365	-6.01163331	-1.2007873
12	-5.9718921	-5.9638814	-1.194378
13	-5.9925151	-5.9976714	-1.198503
14	-5.9792256	-5.9759032	-1.1958451
15	-5.9877901	-5.9899314	-1.197558
16	-5.982271	-5.9808913	-1.1964542
17	-5.9858282	-5.9867178	-1.1971656

18	-5.9835375	-5.9829656	-1.1967075
19	-5.9850131	-5.9853826	-1.1970026
20	-5.9840632	-5.9838258	-1.1968126
21	-5.9846748	-5.9848285	-1.196935
22	-5.9842785	-5.9841794	-1.1968557
23	-5.9845363	-5.9846016	-1.1969073
24	-5.9843691	-5.9843272	-1.1968738
25	-5.9844783	-5.9845066	-1.1968957
26	-5.9844074	-5.9843905	-1.1968815
27	-5.984453	-5.9844644	-1.1968906

This is the real value of the iteration in different type of all value in α. The result is -5.9844.

REFERENCE

i. W.V. Petryshyn; T.E Williamson Jr. A necessary and sufficient condition for the convergence of the sequence of iterates for quasi – nonexpansive mappings, Bull. Amer. Math Soc. 78 (1972), 1027-1031.

ii. M.K. Ghosh and L. Debnath, Approximating common fixed point of families of quasi – nonexpansive mappings, Internet. J. math. and math. Sci. vol. 18, No.2 (1995), 287-297.

iii. B.E. Rhoads and Seyit Femir, Convergence theorems for I– nonexpansive mappings, Internet. J. math. and math. sc. vol. (2006), 1-4.

iv. P. Kuman, W. Kumethong and N. Jewwaiworm, weak convergence theorems of three step Noor interactive scheme for I. quasi – nonexpansive mappings, in Branch space, Appl. Math. sc. Vol2, No. 59(2008), 2915-2920.

v. W.R. Mann, Mean value methods n iteration, proc. Amer. Math, Soc 44 (1952) So6-510.

vi. S.L Singh, C. Bhattnagar and S.N. Mishra, Stability of Junck – type iterative procedures, IJMMS. 19 (2005) 3035-3043.

vii. G.Junck, common fixed points for commuting and compatible maps on compacta, Pro.Amer. Math. Soc. Vol. lo3 no. 3 (1988) 977-983.

viii. G.Junck, commuting mapping on fixed points, Pro.Amer. Math. Monthly 83 (1976), No 4,261-263.

ix. J. Biazar, A. Amirteimoori, An improvement to the fixed point iterative method Appl. Math Comput. 182 (2006), 567-571.