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Economic Production Quantity (EPQ) Model with Time-Dependent Demand and Reduction Delivery Policy

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ABSTRACT

In this paper, production inventory model for time dependent and cost reduction delivery policy is considered. Production rate is considered to greater than demand rate. Mathematical model is presented to find optimal order quantity and total cost. The main objective of this work is to derive optimum quantity to investigate the effect of reduction of cost delivery policy in the inventory model. Numerical example is provided to validate the optimal solution. In addition sensitivity analysis is carried out to analyze the effect of variations in the optimal solution with respect to the change in one parameter at a time.

Keywords: Production, *inventory; optimality; time dependent production rate; time dependent demand rate*

1. INTRODUCTION

The main aim of inventory management is to maximize the total inventory profit or to minimize the total inventory cost. The EOQ (Economic Order Quantity) and EPQ (Economic Production Quantity) are used for finding the optimal order quantity. In this paper demand rate and production rate both considered to be time- dependent. The EOQ models consider that the order for an item is received at one fixed time, in which the EPQ models assume that the item are produced and added into inventory than all at the same time as EOQ model. The objective of this work is to investigate the effect of total cost with the variation of key parameters.

Harries [1] studied EOQ (Economic Order Quantity) model and derived some useful results. After that Taft [2] presented EPQ (Economic Production Quantity) formula. Many inventory models in the inventory literature were presented considering time- dependent demand. Silver and Meal [3] were first presented an EOQ model for the case of the time varying demand. Donaldson [4] developed an inventory model with linearly time dependent demand. Teng *et al*. [5] established inventory model under trade credit financing with increasing demand. Khanra *et al*. [6] presented an inventory model for deteriorating item with time- dependent demand. Teng *et al.* [7] developed an EOQ model for increasing demand in a supply chain with trade credits. Large number of research papers presented by authors like Dave and Patel [8] , Jalan and Chauduri [9] , Jalan *et al*. [10] , Mitra *et al.* [11] in this direction.

Samanta and Roy [12] established a continuous production inventory model of deteriorating item with shortages and considered that the production rate is changed to another at a time when the inventory level reaches a prefixed level. Hou [13] established an EPQ model with imperfect production process, in which the setup cost is a function of capital expenditure. Cardenas- Barron [14] presented an Economic Production Quantity (EPQ) inventory model with planned backorders for deteriorating the Economic Production Quantity for a single product. Cardenas- Barron [15] established an EPQ model with production capacity limitation and breakdown with immediate rework. Krishnamoorthi *et al.* [16] established a single stage production process where defective item produced are reworked and two models of rework process are considered , an EOQ model for with and without shortages. Recently, Liu et al. [17] established the problem of a production system that can produce multiple products but also subject to preventive maintenance at the setup times of some products. Taleizadeh *et al*. [18] developed a vendor managed inventory for two level supply chain comprised of one vendor and several non- competing retailers, in which both the raw material and the finished product have different deterioration rates. Ghiami and Williams [19] established a production- inventory model in which a manufacturer is delivering a deteriorating product to retailers.

The rest of the paper is organized as follows. Assumptions and notion is mentioned in section 2. In the next section 3, we provide mathematical formulation. In section 4 optimal solutions is discussed. Numerical example is given in section 5.Sensitivity analysis with the variation of different key parameters is discussed in section 6.Finally, conclusion and future research directions are detailed in the last section 7.

2. ASSUMPTIONS AND NOTATION

The following assumptions are made throughout the manuscript:
(i). Items are produced and added to the inventory.
(ii). The lead time is zero.
(iii). Two rates of production are considered.
(iv). No shortages are allowed.
(v). The production rate is always greater than demand rate.
(vi).The production rate is proportional to demand rate.
(vii). Demand rate is time dependent i.e. $D(t)= a + b.t$, where $a > 0, 0\ \ , 0 < b < 1$

In addition the following notations are used to form the inventory models:

$I(t)$	inventory level at any time 't'
P	production rate in units per unit time
D	demand rate in units per unit time
Q_1	on hand inventory level during $[0,T_1]$
Q	production quantity
Q^*	optimal production quantity
p	production cost per unit
h	holding cost per unit time
c	setup cost per setup
HC	holding cost per supplier
TC	total cost
TC^*	optimal total cost
T	cycle time
T_1	production time
PC	production cost
SC	setup cost

3. MATHEMATICAL FORMULATION

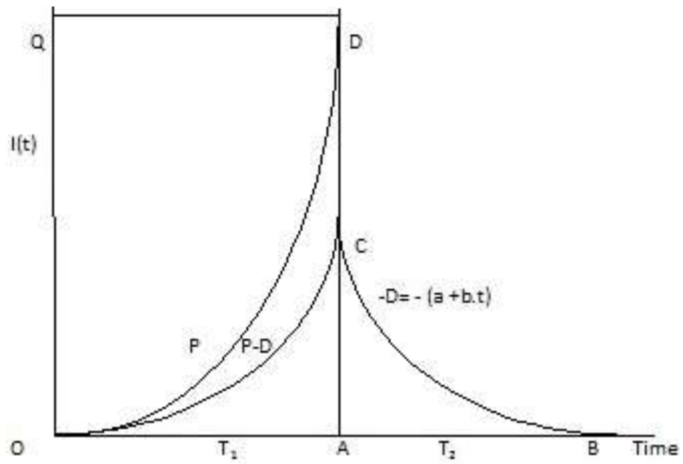

Fig. 1 Inventory vs time

The production starts from O and finished at C. During $t = 0$ to $t = T_1$, production rate is P and demand rate is D. In between $t = 0$ to $t = T_1$ the inventory accumulates at a rate $P – D$. The consumption starts from $t = T_1$ and finished at $t = T_2$. The rate of change of inventory between $[T_1, T_2]$ is given by

$$\frac{dI(t)}{dt} = P - D \quad , \quad 0 \leq t \leq T_1 \tag{1}$$

and
$$\frac{dI(t)}{dt} = -D \quad , \quad T_1 \leq t \leq T_2 \tag{2}$$

where $D \equiv D(t) = a + b.t$, $P = \lambda D$,and $\lambda > 1$.

Solution of (1) and (2) with the condition $I(0) = 0, I(T_1) = Q_1,$, $I(T) = 0$, and $T = T_1 + T_2$ is given by

$$I(t) = (\lambda - 1)\left(a + \frac{bt}{2}\right)t \quad , 0 \leq t \leq T_1 \tag{3}$$

And
$$I(t) = (T - t) + \frac{b}{2}\; T^2 - t^2 \quad , T_1 \leq t \leq T_2 \tag{4}$$

At $t = T_1$ (3) and (4) are same i.e.

$$Q_1 = \;\lambda - 1\;\left(a + \frac{bT_1}{2}\right)T_1 = aT_2 + \frac{bT_2}{2}\; T_2 + 2T_1 \tag{5}$$

At $t = 0$, the order quantity
$$Q = aT + \frac{bT^2}{2} \tag{6}$$

or
$$2aT + bT^2 - 2Q = 0$$

or
$$T = \frac{\sqrt{a^2 + 2bQ} - a}{b} \tag{7}$$

The total cost is calculated by considering setup cost, production cost and inventory holding cost:

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1. Setup cost $SC = \dfrac{c}{T} = \dfrac{bc}{\sqrt{a^2+2bQ}-a}$ (8)
2. Production cost $PC = \dfrac{p}{2}\; a+\sqrt{a^2+2bQ}$ (9)
3. Inventory holding cost $HC = \dfrac{h}{T}\left[\int_0^{T_1}(\lambda-1)\left(a+\dfrac{bt}{2}\right)t.dt+\int_{T_1}^{T_2}\left\{a(T-t)+\dfrac{b}{2}\;T^2-t^2\right\}\mathrm{dt}\right]$

$$=\frac{h}{2T}\left[(\lambda-1)\left(a+\frac{bT_1}{3}\right)T_1^2+a(T_2^{\,2}-T_1^{\,2})+\frac{b}{3}(T_2-T_1)(2T_1+2T_2+5T_1T_2)\right] \quad (10)$$

Total cost is *TC= SC + PC + HC*

$$=\frac{b(c+Ah)}{\sqrt{a^2+2bQ}-a}+\frac{p}{2}\;a+\sqrt{a^2+2bQ} \quad (11)$$

Where $\dfrac{1}{2}\left[(\lambda-1)\left(a+\dfrac{bT_1}{3}\right)T_1^2+a(T_2^{\,2}-T_1^{\,2})+\dfrac{b}{3}(T_2-T_1)(2T_1+2T_2+5T_1T_2)\right]$

4. OPTIMAL SOLUTION

The optimal solution is obtained by differentiating (11) with respect to Q , we get

$$\frac{dTC}{dQ}=-\frac{b}{\sqrt{a^2+2bQ}-a^{\;\;2}\;\;a^2+2bQ^{\;1/2}}\left[b(c+Ah)-p\;\;\sqrt{a^2+2bQ}-a^{\;\;2}\right] \quad (12)$$

And $\dfrac{d^2TC}{dQ^2}>0$ (13)

The optimal (minimum) production quantity Q = Q* is obtained by solving $\dfrac{dTC}{dQ}=0$, we get

$$Q=Q^*=\frac{\left[b(c+Ah)+2a\left\{\dfrac{b(c+ah)}{p}\right\}^{1/2}\right]}{2b} \quad (14)$$

5. NUMERICAL EXAMPLE:

Let us consider the parameter values of the inventory system a = 5000 unit/year, λ =1.3 , b = 0.2 unit/year , c = 100 , h = 10 per unit / year , p = 100 units , T_1 0.2 year, T_2 = 0.25 year. Substituting these values in (13) and (10), we get Q = Q*= 25567.1and TC = TC*= $ 25100.3.

6. SENSITIVITY ANALYSIS

Sensitivity analysis is performed with the variation of different key parameters. Taking all the numerical values as mentioned in the above numerical example.

Table 1: Variation of production cost 'p', setup cost 'c' and holding cost 'h' on the optimal solution

p	*Q*	*TC*	*c*	*Q*	*TC*	*h*	*Q*	*TC*
110	24389.2	26325.3	110	25817.8	25101.3	11	26312.4	25103.2
120	23361.8	27495.9	120	26066.3	25102.3	12	27037.7	25105.9
130	22455.4	28618.6	130	26312.4	25103.2	13	27744.7	25108.6
140	21647.9	29698.8	140	26556.3	25104.1	14	28434.6	25111.3
150	20922.5	30741.1	150	26798.1	25105.0	15	29108.7	25113.9

All the above observations can be sum up as follows:

- Increase of production cost leads decrease in order quantity and increase in total cost
- Increase in setup cost leads increase in order quantity and slight increase in total cost
- Increase in holding cost leads increase in order quantity a slight increase in total cost

7. CONCLUSION

In this paper, we developed production inventory model with time- dependent demand rate. Mathematical formulation is presented for finding optimal order quantity and total cost. We have also shown that the total cost is convex function with respect to order quantity. A numerical example is provided to demonstrate the applicability of proposed model. Sensitivity analysis shows that the changes are quiet sensitive with the change of parameters.

This model discussed can be extended for several ways. We may extend the model for stock- dependent demand. We could generalize the model for deteriorating items.

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